10.26.22

LECTURE 27

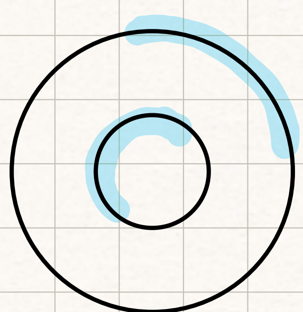
curving of a curve is reflected by changing direction of a tangent vector → rate of change of tan. vector

Definition

if C is a curve, and $r(s)$ is an arc-length parameterization, the curvature $K(s)$

$$K(s) = \|T'(s)\|$$

- b/c $r(s)$ is an arc-length param, $\|T'(s)\| = \|r''(s)\|$
- $K(s)$ does not depend on the arc-length param
- measures rate of change of $T(s)$ wrt arc length
  - ↳ how does $T(s)$ change when arc length is constant?



is constant, but curvature is much different

Theorem

$r_1(t)$ is a smooth param of C, then suppose $P = t_0$ and P lies on C, then

$$K(t=t_0) = \frac{\|T_1'(t_0)\|}{\|r_1'(t_0)\|}$$

Theorem

$$K(t) = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3}$$

both ways to calculate $K(t)$

EXAMPLE

compute $K(t)$ for $r(t) = \langle A\cos(t), A\sin(t), 5\rangle$ for $0 \le t \le 2\pi$, $A > 0$

$$K(t) = \frac{\|T'(t)\|}{\|r'(t)\|} \qquad T'(t) = \frac{r'(t)}{\|r'(t)\|}$$

$$r'(t) = \langle -A\sin t, A\cos t, 0\rangle$$

$$\|r'(t)\| = \sqrt{A^2\sin^2 t + A^2\cos^2 t + 0^2} = A$$

$$T'(t) = \langle -\sin t, \cos t, 0\rangle$$

$$\|T'(t)\| = \sqrt{\sin^2 t + \cos^2 t} = 1$$

$$K(t) = \frac{1}{A}$$

big radius circles have small curvature

LECTURE 27 PROBLEMS

1) B

2) $K(t)$ of $r(t) = \langle 3\sin t, -3\cos t, 4t\rangle$

$$K(t) = \frac{\|T'(t)\|}{\|r'(t)\|} \qquad T'(t) = \left\langle \frac{3}{5}\cos t, \frac{3}{5}\sin t, \frac{4}{5}\right\rangle$$

$$r'(t) = \langle 3\cos t, 3\sin t, 4\rangle$$

A

$$\|r'(t)\| = \sqrt{3^2\cos^2 t + 3^2\sin^2 t + 4^2} = 5$$

3) $r'(t) = \left\langle \frac{4}{5}\cos(e^t), \sin(e^t), \frac{3}{5}\cos(e^t) \right\rangle$

$$\|r'(t)\| = \sqrt{\frac{16}{25}\cos^2(e^t) + \sin^2(e^t) + \frac{9}{25}\cos^2(e^t)}$$

$\|r'(t)\| = 1$

$T'(t) = r'(t)$ and $\|T'(t)\| = 1$

$K(t) = 1$

4) A